**King's College London (KQC) University of London**

# SCIENCE SIMULATIONS LABORATORY

## NEWTON'S SATELLITE ORBITS

## STUDENTS' MANUALS (Version 1.02.2003)

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## STUDENT MANUAL A – IT’S AN OLD IDEA

If you throw a ball straight out in front of you, horizontally, it will travel a few metres before it hits the ground. If you throw it faster it will travel further before it lands. What’s the greatest distance it could possibly travel before landing? Could it even go in a circle right round the Earth – if only it could be launched fast enough?

Actually this problem has been discussed for some time. Here is an extract from Sir Isaac Newton’s book, ‘The system of the World’, first published, in Latin, in 1687.

“.... The greater the velocity is with which (a stone) is projected, the farther it goes before it falls to the earth. We may therefore suppose the velocity to be so increased, that it would describe an arc of 1, 2, 5, 10, 100, 1000 miles before it arrived at the earth, till at last, exceeding the limits of the earth, it should pass into space without touching it.

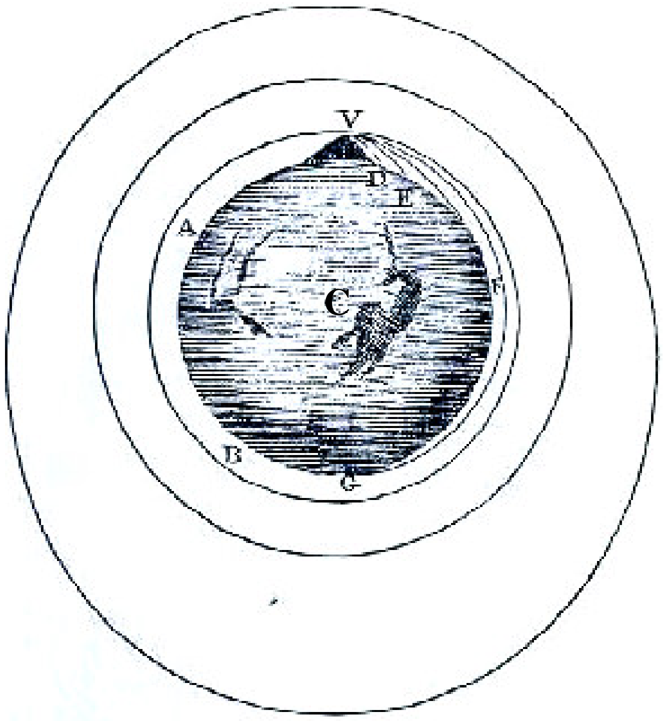


Figure A1

Let AFB represent the surface of the earth, C its centre, VD, VE, VF the curved lines which a body would describe, if projected in a horizontal direction from the top of a high mountain successively with more and more velocity; ...let us suppose either that there is no air about the earth, or at least that it is endowed with little or no power of resisting; the body projected with a less velocity describes the lesser arc VD, and with a greater velocity the greater arc VE, and, augmenting the velocity, it goes farther and father to F and G, if the velocity was still more and more augmented, it would reach at last quite beyond the circumference of the earth, and return to the mountain from which it was projected”.

Newton did not actually try launching a series of bodies with greater and greater velocity from a mountaintop to see what would happen. He could not project them fast enough to travel one or two, let alone 100 or 1000 miles. Instead he thought about what would happen if anybody could actually try the experiments, and his conclusions were as described in the quotation.

In some ways we are better off today than Newton was: a real experiment not unlike the one he thought about is carried out when an Earth satellite is put into orbit. The satellite is launched more or less vertically upwards, and then, when it reaches an appropriate height, it is ‘injected’ into orbit with a horizontal velocity. Whether or not the satellite will go into orbit, and if so what orbit, depends on the height and injection velocity.

Although you cannot actually do the experiment, you can use the computer to work out what would happen if you did do it. A series of equations have been built into the computer program called ***Newton’s Satellite Orbits*** which enables the computer to calculate the position and velocity of the satellite every 100 seconds. The basic law of motion F = ma is one of these equations, F being the Earth's gravitational pull which is always directed towards the centre of the Earth. This force varies with distance from the centre of the Earth according to the 'inverse square law'.

You will be able to use the computer program to answer questions like these:

> What velocity is needed to make the satellite travel in a nearly circular orbit close to the Earth's surface?
>
> How long does it take to go round once?
>
> Does its speed change as it goes round?
>
> What happens if the satellite is 'injected' with a larger velocity?

But before using the computer to answer these questions and many others you will need to think about some easier questions in Manual B.

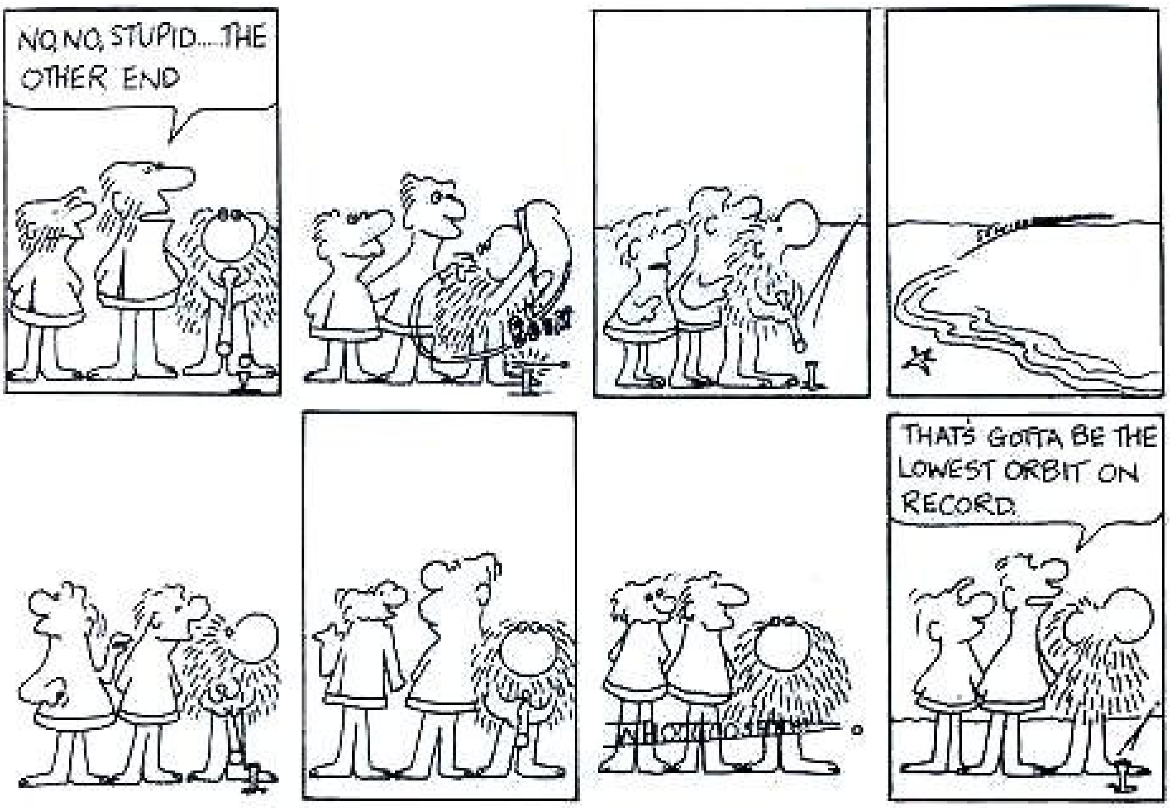
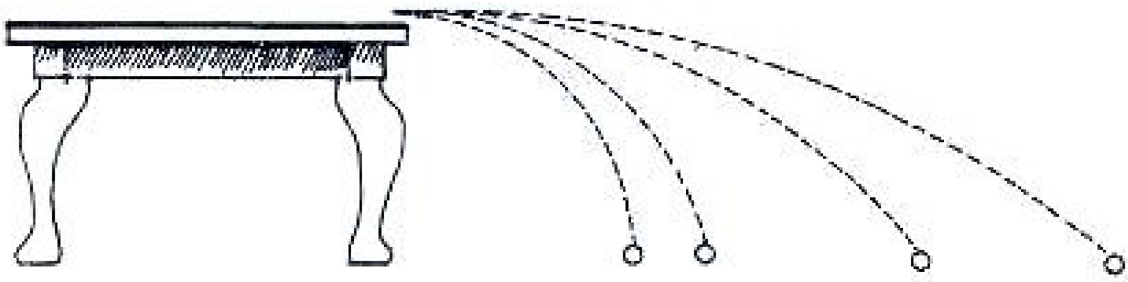



Figure A2

## STUDENTS MANUAL B – TRAJECTORIES AND ORBITS

The diagram shows the paths of four balls launched horizontally from the edge of a table.



This shows a rather small-scale experiment, and so the surface of the Earth was drawn flat. Figure B1 represents a mountain on the Earth's surface; now the curvature of the Earth shows up.

*B1 What is the direction of the Earth's pull on the body at B?*

All three bodies are moving horizontally (parallel to the Earth's surface) when they start off. C continues to move in the same direction and so gets further and further away from the Earth.

*B2 Could this really happen? Under what conditions?*

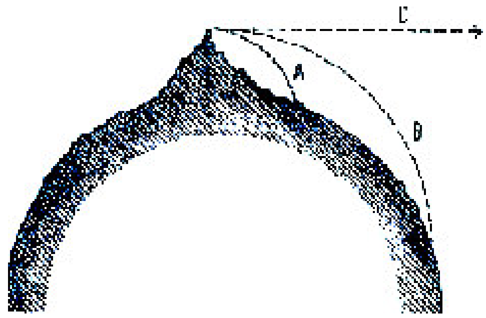



Figure B1 Launch from a mountaintop

Before using the computer you should do some approximate calculations to find out what sort of launch velocity is needed to put a body into orbit just above the Earth' surface.

*B3 You know that for a body undergoing uniform acceleration $s = ut+½at^2$. Rearrange this relationship to get an expression for the time it takes a body to fall a distance, s, if it is dropped from rest. t = ?*

Imagine an enormous flat table resting on one point of the Earth' surface. Because the Earth is round the ground gets further and further below the table as you move away from the point of contact. In fact by the time you have gone 6 km. along the table the Earth is about 3 m below you (figure B2).

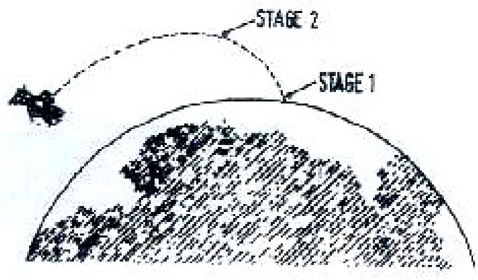

Figure B2 Curvature of the Earth is about 3 m in 6 Km.

So if a probe launched horizontally is going to go into orbit just above the Earth's surface it must fall about 3 m in the time that it travels forward 6 km.

*B4 Use your answer to B3 to calculate the time it will take a body to fall 3 m, with an acceleration of 10 m s $^{-2}$.*

*B5 How fast must the probe be launched if it is to travel forward 6 km in this time?*

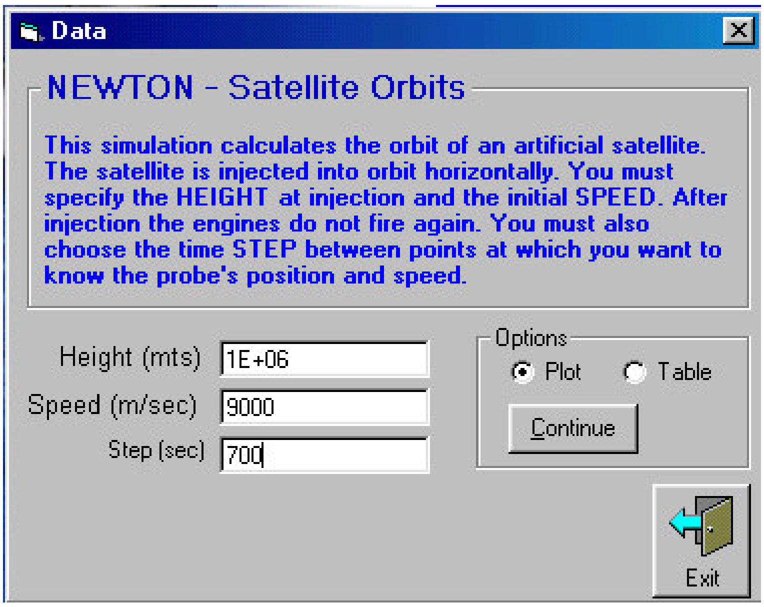
## STUDENTS' MANUAL C – INTO ORBIT




When an Earth satellite is launched its rocket engines fire in several stages. Stage 1 gets it up to the required height and then Stage 2 gives a big horizontal velocity (i.e. parallel to the Earth's surface) to 'inject in into orbit'. Calculating this injection velocity is critical: small changes in velocity may lead to big changes in the orbit.

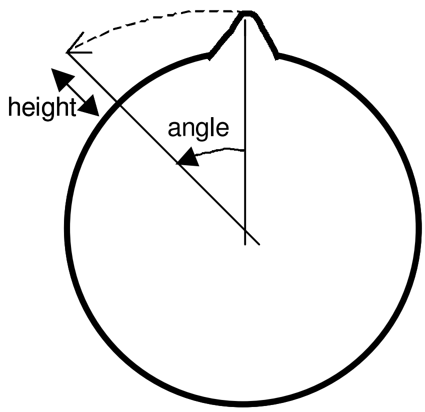
The last question on Manual B should have given you some idea of a sensible value to try for the injection velocity, if the satellite is to stay at about the same height above the Earth's surface. Now try using the computer simulation Newton's Satellite Orbits to see if you can launch a satellite into orbit horizontally so that it will neither 'splash down' nor, like C in Figure B1 of Manual B, get further and further away from the Earth.

You will have to choose the HEIGHT at the 'injection' point, and the injection SPEED. The computer will calculate the body's position after 100, 200, 300... seconds, but you must decide the time STEP between successive outputs.




After the results have been displayed you can rerun the program using different values of HEIGHT, SPEED of STEP.

To answer some of the questions that follow you will need more detailed information than you have from a PLOT of the kind shown above. The option TABLE gives you values for the height of the satellite above the Earth's surface, its speed and the angle is has travelled since injection.



When you have succeeded in putting your body into orbit you ought to be able to answer the following questions:

*C1* *What time is need for an artificial satellite to go once around the Earth, close to the surface?*

*C2* *What do you think happens to it after it returns to its starting point?*

*C3* *What happens to the orbit if the probe is given a velocity slightly higher than is needed to make it go into orbit?*

*C4* *At what point in the orbit is the probe at (a) its maximum height (b) its minimum height? What shape is the orbit?*

*C5* *From your answers to the last question you should be able to say where the probe has (a) its maximum (b) its minimum potential energy.*

*C6* *Once the satellite has been launched no engine or rocket on it is fired –so its total energy is constant. Use this fact, and your answer to C5 to say where the satellite must have (a) most (b) least kinetic energy.*

Now look at your results from the computer: do the maximum and minimum speeds occur at the positions you predicted in your answer to C6?

*C7* *In your physics lessons you have been taught that the path followed by a projectile is a parabola. Next time you see a football match have a close look at the path followed by the ball after a goal kick. Is it parabolic? (If so it should be symmetrical – coming down at the same angle as it goes up). If not –how does it differ, and what do you think causes the difference between theory and practice?*

The last question illustrates a typical aspect of physics – the situations which it describes are often simplified versions of real ones: footballs do not move in exact parabolas; real gases do not obey the 'ideal gas' laws precisely; the period of a simple pendulum is not completely independent of the amplitude; but it is only by making certain simplifications that we can make any useful progress at all, and in many cases the simplified theory does give results quite close to what actually happens.

*C8* *What is the simplified theory on which the computer 'experiment' is based, and how does it differ from reality? How do you think the results of a real experiment might differ from the predictions made by the computer program?*

As Manual A pointed out a real experiment, not unlike the one Newton thought about, is carried out every time an Earth satellite is put into orbit. Compare your computed values with some of the data for real satellites given in Manual D.

## STUDENTS' MANUAL D –GOING FURTHER

You will have learnt from this module that you do not have to increase the injection velocity very much to make a big difference to the size of the orbit. In practice, for example to start a space probe on its way to the Moon, the third stage of the rocket is used. It produces further acceleration, and depending on the speed it gives, puts the probe into an orbit which will carry it towards the Moon or the distant planets.

This program is not really suitable for finding out what happens at higher velocities or when the probe gets further and further away from Earth. To find out what launch velocity is needed to go as far as the Moon or the sun you will need to learn what the term 'escape velocity' means through the use other calculations and computer programs.