King's College London (KQC) University of London

SCIENCE SIMULATIONS LABORATORY

PARTICLE SCATTERING

STUDENTS' MANUALS (Version 1.02.2003)

Author: J. Harris
Programmer: G. Page (1982 GW Basic Version)
D. Terry (2003 Visual Basic Version)
Editors: J. Harris, (1982 Version), C. Michelsen M.A. Ph.D. (2003 Version)

STUDENTS' MANUAL A - SCATTERING BY HARD OBJECTS - A COMPUTER SIMULATION

The scattering of alpha particles by thin metallic foils early in this century helped confirm Rutherford's idea that the atom has a small massive charged nucleus. Today physicists bombard various targets with fast moving elementary particles to probe the structure of matter.

It is a little bit like trying to find out something about the size and shape of something you cannot see by observing how marbles bounce off it.

The three computer programs in this unit simulate different experiments – from marbles scattered by hard targets of various shapes to alpha particles scattered by metallic foils.

In the first simulated experiment the bombardment is by a hail of marbles projected in a plane (say across the surface of a table) and moving parallel to one another. The marbles are randomly spaced in a beam 10 units wide. The targets are placed symmetrically in the middle of the beam and may provide a square, circular or triangular barrier to the beam. (Figure A1). The targets may be from 1 unit to 10 units wide. They are 'hard' and 'massive', i.e. a bombarding particle loses no energy in a collision.

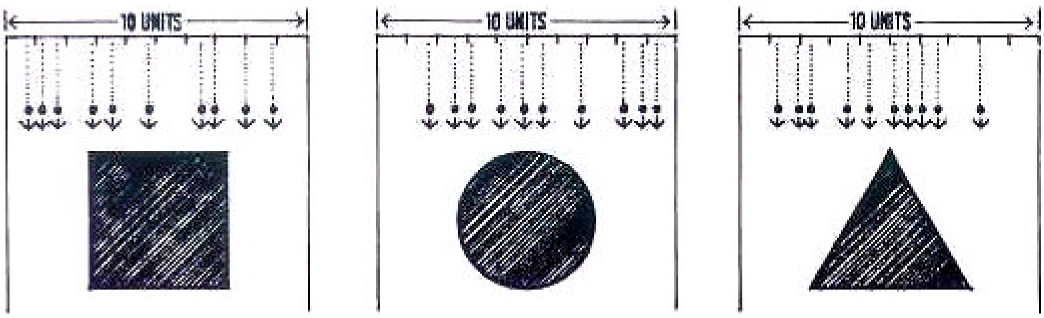


Figure A1 A hail of randomly spaced particles probes a target which may be square, circular or triangular.

The scattering angle ϕ for each particle is calculated from the angle at which the particle hits the surface of the target, making the usual assumption that the angles of incidence and reflection are equal.

Note that if a particle travels straight on without being deflected the scattering angle is 0°; a particle turned back in its tracks is scattered through 180°. (Figure A2).

Figure A2 Scattering angles of (a) 0°, (b) 90°, (c) 180°

The computer will show you the tracks of 10 particles before and after being deflected by the target; the target itself remains hidden (Figure A3).

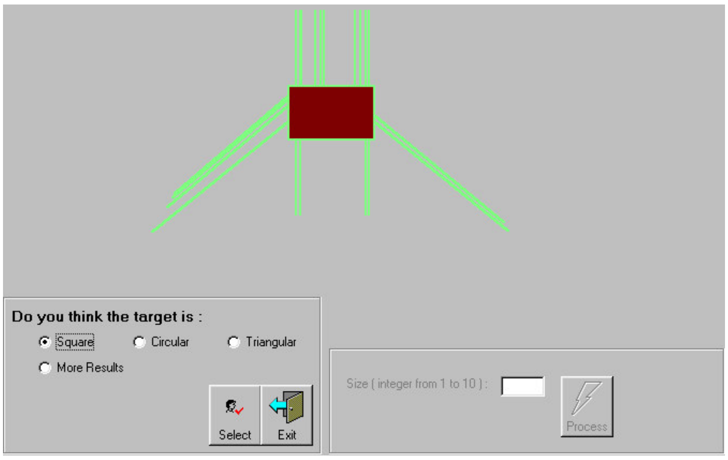


Figure A3 Screen printout of SCATT1.

Before turning to the computer think about the results you might expect to get with different targets.

A1 *Suppose the target were a square, side 3 units long (Figure A4).*

a *Out of 10 particles how many would you expect to be undeflected?*

b *Through what angle(s) would the others be deflected?*

A2 *Sketch the result you might expect if the object were:*

a *Triangular*
b *Circular*

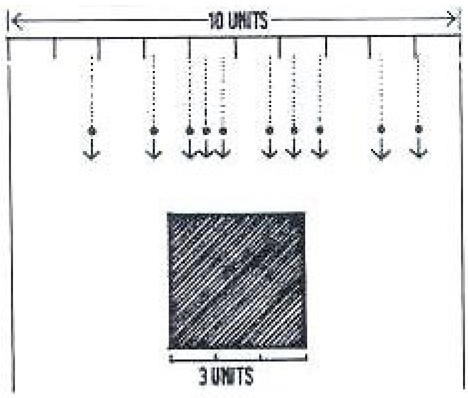


Figure A4 10 particles are scattered by a square target 3 units wide.

Now run the program SCATT1 several times.

In these simulated scattering experiments you can see the particles' tracks before and after scattering.

A3 *Suppose you were only told that out of a total of 10 randomly spaced particles six were deflected through 60° and four were undeflected?*

a *What shape do* you think the *target was?*
b *What is your* estimate *of its size?*

STUDENTS' MANUAL B - HARD AND SOFT TARGETS

In the first simulated experiment the scattering targets were assumed to be massive and hard. In this context 'massive' means that any velocity a target may acquire, on impact, can be neglected – we assume it doesn't move; 'hard' means that the target is solid, undeformable. Both these conditions affect the scattering of the particle.

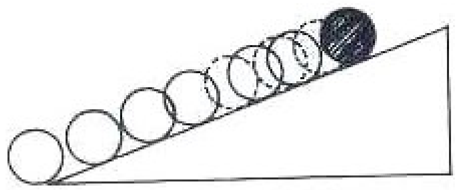
B1   *a*   *What difference does 'massive' make? Imagine a cylindrical target standing upright on a table; the particle moves towards it across the table.*

How will the angle through which a particle is scattered change if the target moves?

(A helpful way to think about questions like this is to consider extreme cases – what if the target were very light compared with the bombarding particle, and vice versa).

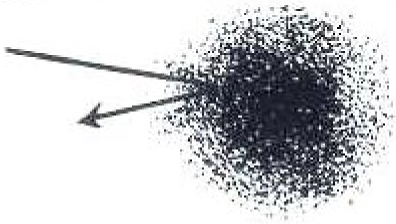
b   *Which assumption (massive or free to move) should be used in a model to represent the scattering of alpha particles (mass 4 amu) by nuclei of metal atoms, say gold (mass 197 amu)?*

B2   *How do the kinetic and potential energies of a ball change as it moves up a slope, slows, stops and returns?*



B3

a   *A particle is moving against an inverse square repulsive force directly towards the centre of the field of force. Sketch a graph to show how the force on the particle varies with the distance of the particle from the centre of the field of force.*



b   *Sketch another force – distance curve for a particle colliding with a massive hard cylinder.*

B4   *Would the angle through which a single particle is scattered depend* on *the energy of the particle for:*

a   *a massive hard cylinder?*

b   *a massive inverse square scatterer?*

It is helpful to think about the time for which the force acts on the particle when estimating the deflection – how long it spends in the force field or how long it is in contact with the cylinder.

STUDENTS' MANUAL C - THE SIZE OF A ' SOFT' TARGET

In the ' experiment' with hard targets (Manual A) you could estimate the target' s size from the fraction of particles which hit it. All particles with an ' aiming error' greater than the target' s radius pass by completely undefected.

The size of an inverse square scatterer is a less obvious notion because there is no clear cut difference between hits and misses. One definition of ' size' is the nearest distance a particle aimed ' head-on' at the target get it before being stopped and turned back (Figure C1). This definition works (agrees with our everyday idea of size) for hard objects too.

C1 The ' size' for an inverse squarescatterer defined in this way is related to the energy of the scattered particle. Explain.

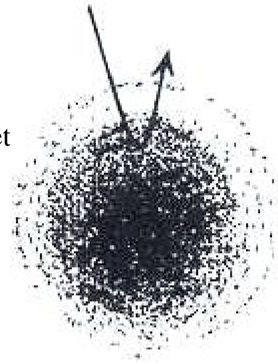


Figure C1 A definition of size for a ' soft' target.

The scattering law for a single massive inverse square scatterer is derived by applying Newton' s second law in the form, change in momentum = ∫Fdt, and the principle of conservation of angular momentum. One can find out how much the particle is deflected by the force exerted on it as it passes close to the target (see, for example, Nuffield Advanced Physics, Teacher' s Guide 5, p.112).

The scattering law derived in this way $\cot\frac{\phi}{2} = \frac{2p}{b}$ is:

Here ϕ is the angle through which the particle is deflected; p is the ' impact parameter' (we previously called this the aiming error) i.e. the distance at which it would pass the centre of the target if it sailed by undeflected (Figure C2); b is a parameter which depends on the constant in the inverse square force law and the energy, E, of the bombarding particle. In fact b is equal to the distance of closest approach in a head on collision for a particle with energy E, i.e. it is the size of the inverse square scatterer as defined above (Figure C1).

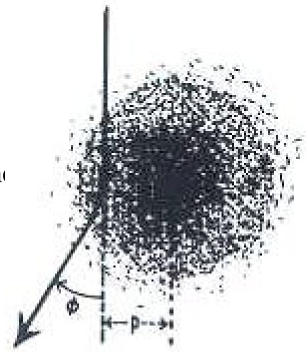


Figure C2 Deflection of a particle (ϕ) depends on ' impact parameter' (p).

C2

a *If we* define ' size' *as being the distance of closest approach in* a *head-on collision, what will cot ϕ /2 be for a particle (with the same energy) aimed at the ' edge' of the target (Figure C3).*

b *What are the corresponding values of 0/2* and 0*? (A graph of the cotangent function is given* in *Manual G).*

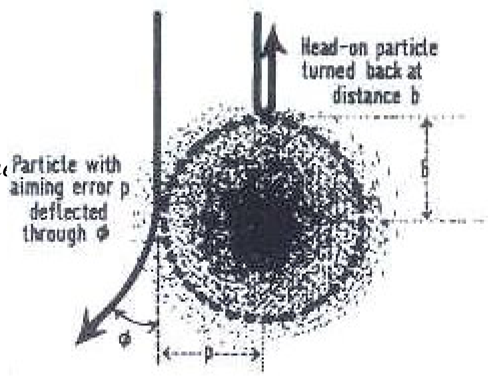


Figure C3 Particle with impact parameter b deflected through angle ϕ.

C3 *If a particle is deflected (by an inverse square scatterer) through less* than the *value of* ϕ you *calculated in* C2 (b) *what can you say about its impact parameter?*

C4 *What will be the value of* ϕ *for a particle aimed outside the edge of a hard cylinder?*

You should now have two ways of telling whether the target is ' hard' (a cylinder), or ' soft' (an inverse square scatterer) from results you will obtain when you try the next ' experiment' SCATT2.

The formula

$$\cot\frac{\phi}{2} = \frac{2p}{b}$$

is used to calculate the scattering angle ϕ for inverse square scatterers. The impact parameter p has a value which depends on the random position of each particle in the beam. For hard cylinders ϕ is calculated from the angle of incidence, which also depends on p of course (Figure C4).

For a hard cylinder $\phi = 180 - 2i$ For a 'soft' target $\cot\frac{\phi}{2} = \frac{2p}{b}$

Figure C4 Scattering by hard and soft targets.

Reference: Nuffield Advanced Physics (1971) Teachers' Guide, Unit 5. Longmans.

STUDENTS' MANUAL D – A SECOND COMPUTER SIMULATION – USING SCATTERING TO DISTINGUISH BETWEEN HARD AND SOFT TARGETS

Before using the second simulation (SCATT2) you should know at least one way of telling whether a target is a hard cylinder or an inverse square scatterer from the results of a scattering experiment. You will also need to understand how the concept of size has been defined for the soft target (look at Manual C again if you are not sure).

In the simulation SCATT2 a target is chosen at random and, as in the first simulation it is bombarded by a randomly spaced hail of particles. The scattering angle is calculated as explained in Manual C (Figure C4).

The computer does not show you the track of each particle as in the first ' experiment' ; instead the angle through which each one is scattered is plotted in a histogram.

A screen printout is shown below. You will notice that you have to specify the energy of the bombarding particles.

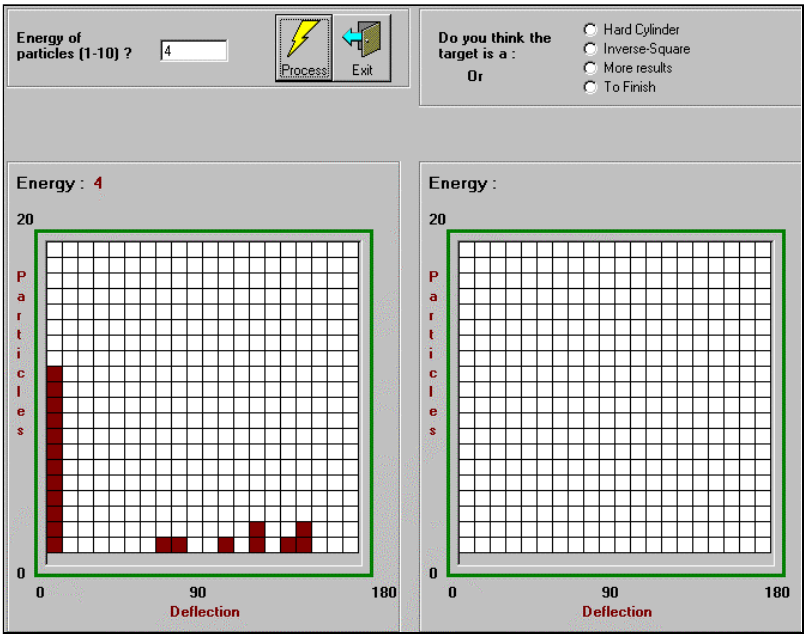


Figure Dl Screen printout of SCATT2

After the scattering angles for 20 particles have been displayed in a histogram you are asked to interpret the results – or you can ask to see more results. If you do ask for more results you have the chance to choose a different energy for the next set; the extra information that this gives should help you determine the nature of the target.

You will eventually be asked to estimate the size of the target. Remember that the size of a soft target depends on the energy of the probing particle.

When you are confident that you can distinguish hard from soft targets from the scattering results go on to the last part of this unit where you are asked to find out what model of the nucleus is used in a simulation of the scattering of alpha particles by metal foils. (Manual E).

STUDENTS' MANUAL E – SIMULATION OF SCATTERING OF ALPHA PARTICLES – A COMPUTER MODEL

In the first two simulations in this unit you have learnt how we can infer something about the shape and size of a target from the scattering pattern it produces. Now we can go on to consider the use of the scattering technique to learn something about the nature of matter, using alpha particles to probe the structure of atoms.

From the earlier work in this unit you should know two characteristics of scattering patterns from which one can distinguish an inverse square scatterer from a hard sphere. If you are not sure of at least one of these, refer back to Manual B.

Of course a real alpha-scattering experiment is more complicated than the situations considered so far, which were concerned with a single target bombarded by a random beam of particles. For example one cannot fire an alpha particle at a single isolated atom.

E1 *Suppose you knew the scattering pattern produced by a single target. How would it change if there were:*

a *2*
b *10*
c *n*

scattering centres in the beam?

E2 *What assumptions have you made in answering the last question? (Hint: Is there any limit to the value of n?)*

In a real alpha scattering experiment the beam is scattered by a thin foil of metal. Clearly the more atoms there are in the path of the beam the greater the chance of an individual alpha particle being scattered. Also, increasing the thickness of the foil increases the chance of a particle being scattered two or more times. The computer program simplifies the problem by ignoring the possibility of multiple scattering. Physically this is equivalent to restricting the experiment to a thin film, and so the simplification is justified under 'thin film' conditions.

When a thin metal foil is bombarded with alpha particles the majority pass through with hardly any scattering. From this we might infer that, as far as the alpha particles are concerned, the atoms composing the foil are mostly empty space. But just occasionally an alpha particle is scattered through a large angle. This is the key observation which led Rutherford to propose his nuclear model of the atom. Only very rarely is an alpha particle travelling close enough to the tiny nucleus to be much affected by it.

In a scattering experiment the factors which can be varied include:

the material of which the foil is made;
the foil thickness;
the velocity (kinetic energy) of alpha particles

E3 *Which factor(s) would you vary* in a *scattering experiment to help you decide between the hard sphere and the inverse square scatterer model of the nucleus?*

The program SCATT3 simulates a scattering experiment. The program is based on a simple model of the atom. By varying the 'experimental' conditions, try to decide whether the model used is an inverse square or hard sphere. To keep your simulation close to possible experiments, make use of the following data:

The energy of alpha particles is about 5 MeV (can be reduced by putting thin layers of mica in the beam);

The thinnest metal foils are about 10^{-8} m thick

The metals usually used are gold, silver, copper or aluminium.

The output in SCATT3 is similar to SCATT2. It tells you the angle through which each alpha particle is scattered. Since very few particles are scattered through large angles only a small range (0 to 15° or 0 to 1.5° – you must choose) is displayed but particles scattered through angles outside the chosen range are recorded as shown in the example.

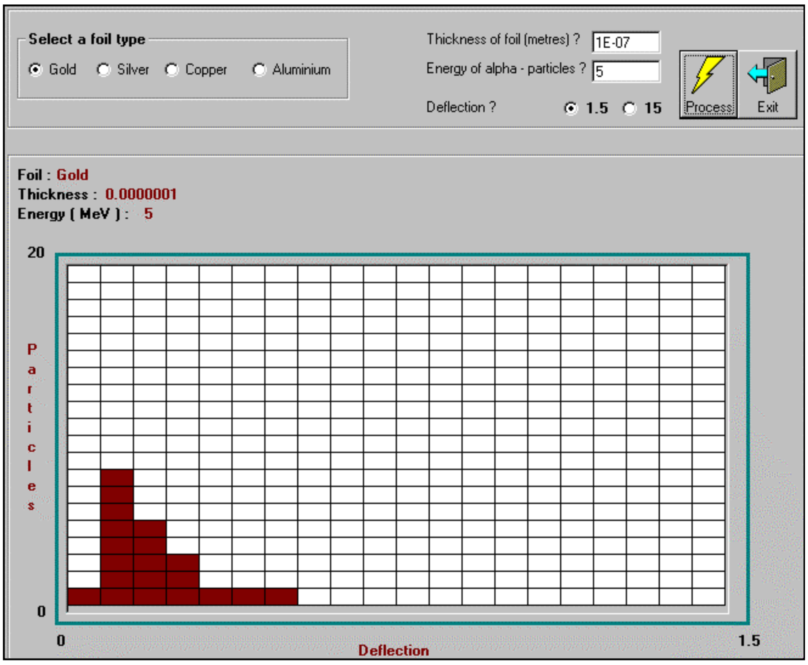


Figure E1 Sample results for SCATT3.

Now use SCATT3 to try to answer some questions.

E4 a *Is the scattering model ' hard sphere' or ' inverse square' ?*

b *Does scattering depend on foil thickness? How? Why? (In the program a random number is generated to determine the impact parameter for every nucleus which the alpha particle passes on its way through the foil. In this ' single scattering' model the smallest impact parameter is chosen since this gives the largest scattering.)*

c *Do some metals scatter more than others? If so, what property(ies) of* the metal *might be involved? Some properties which might be relevant are listed below:*

Properties of metals

	Density /10^3 kg m^{-3}	*Atomic radius /10^{-10} m*	*Atomic weight /amu*	*Atomic number*
Aluminium	*2.65*	*1.43*	*27*	*13*
Copper	*8.9*	*1.28*	*63.5*	*29*
Silver	*10.5*	*1.44*	*108*	*47*
Gold	*19.3*	*1.44*	*197*	*79*

STUDENTS' MANUAL F - HOW GOOD IS THE MODEL?

By now you will have decided which model was used to construct the computer program SCATT3, and you may have learnt something about the predictions it makes for the results of real experiments. This is the real test of a model of course – can it explain or predict experimental observations?

These results are from a paper by Geiger (1910):

a For gold foils about 1 x 10^{-7} m thick, bombarded with 5.5 MeV alpha particles, the most probable scattering angle increased with the number of gold foils is as follows:

N° of foils (each about 10-7 m)	Most probable scattering angle
1	0.17°
2	0.38°
4	0.60°

b If the velocity of the alpha particles was reduced to about half of its initial value the range of angle into which they were scattered became much greater and the most probable scattering angle for a single foil increased to about 1.3°.

c With two gold foils the probability of different scattering angles varied as shown in Figure Fl. (Each foil about 10^{-7} m thick; alpha particle energy 5.5 MeV).

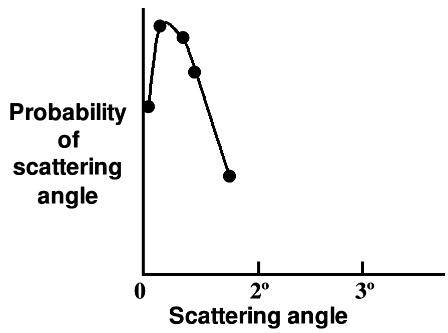


Figure F1 Probability of various scattering angles for 2 gold foils.

F1 (For discussion) Are you now prepared to accept this model for the atom? If not, what reservations do you have and what else would you want to do?

REFERENCE

Geiger, H., (1910) The scattering of alpha particles by matter. Proc. Roy. Soc. 83, 492.

STUDENTS' MANUAL G – GRAPH OF COTANGENT AGAINST ANGLE